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Non-factorizable contribution to $\bar{B}^0 \to D^0\pi^0$

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Abstract. For charmed color-suppressed $\bar{B}^0 \to D^0\pi^0$ decay, non-factorizable contributions are expected to be leading, and the naive factorization description breaks down. We study the $1/m_b$ power-suppressed non-factorizable effect in $\bar{B}^0 \to D^0\pi^0$, which is due to soft exchange between the emitted heavy–light quark pair and the $B\pi$ system, in the framework of QCD light-cone sum rules. The resulting correction to the decay amplitude is found to be numerically comparable with the corresponding factorizable piece, estimated to be at about (50–110)% of the latter. The relevant parameter a_2 receives a positive number contribution, due to the factorizable correction and the power-suppressed soft effect. Our findings could be crucial to a phenomenological understanding of the $\bar{B}^0 \to D^0\pi^0$ decay.

1 Introduction

"Naive" factorization [1], or the "generalized" factorization [2] developed subsequently, has been viewed as a simple but predictive model for two-body hadronic decays of heavy mesons prior to the presentation of QCD factorization [3]. At present, it is known to us that for a large class but not all of two-body non-leptonic B decays, QCD factorization can furnish a rigorous theoretical basis for the naive factorization assumption of the hadronic matrix elements. Some examples for which the naive factorization holds up to power corrections in $\Lambda_{\mathrm{QCD}}/m_b$ and α_{s} are the charmless decays $B \to \pi\pi, \pi K$ and the class-1 charmed decays $\bar{B}^0 \to D^{(*)+}\pi^-$ (relevant to the parameter a_1). In the heavy quark limit $m_b \to \infty$, but to all orders of perturbation theory, this type of processes can be systematically computed in terms of convolutions of hard-scattering kernels with the leading-twist light-cone distribution amplitudes of the corresponding light mesons. Such a treatment is based on a color-transparency argument [4] that for the limit in question the momentum carried by a light meson which is directly emitted off the relevant weak vertex is so large that it has not sufficient time to exchange soft gluons with the system including the decaying B meson and the produced meson picking up a spectator quark. Typical examples for which QCD factorization does not apply are the class-2 charmed decays $\bar{B}^0 \to D^{(*)0}\pi^0$ (usually called color-suppressed decays for the reason that the relevant phenomenological parameters $a_2(D\pi)$ is of $\mathcal{O}(1/N_c)$ in the large N_c accounting [5]). The reason is that unlike the aforementioned case where a light meson is emitted, the heavy $D^{(*)0}$ meson, as an emitted particle which is neither small ($\sim 1/\Lambda_{\mathrm{QCD}}$) nor fast, is produced in the color-suppressed decays and so cannot be decoupled from the $B\pi$ system. This indicates clearly that the factorization contributions to $\bar{B}^0 \to D^{(*)0}\pi^0$ do not provide a leading result, for all that they are present, and non-factorizable soft contributions, for example the charge-exchanging rescattering processes [6] from the dominate class-1 channel, dominate in such decays. Hence, at present a theoretical understanding is not accessible for the color-suppressed decays $\bar{B}^0 \to D^{(*)0}\pi^0$ and also for the class-3 decays, say, $B^- \to D^{(*)0}\pi^-$ to a certain extent, although in the latter case the class-2 amplitudes are predicted to be power-suppressed with respective to the corresponding class-1 ones in QCD factorization. It is quite a challenge to give a consistent theoretical explanation of the data on $B \to D^{(*)}\pi$, after new experimental observations $\mathcal{B}(\bar{B}^0 \to D^0\pi^0) = (2.74^{+0.36}_{-0.32} \pm 0.55) \cdot 10^{-4}$ [7] and $\mathcal{B}(\bar{B}^0 \to D^0\pi^0) = (3.1 \pm 0.4 \pm 0.5) \cdot 10^{-4}$ [8] have been announced respectively by the CLEO and Belle Collaborations. While there exist attempts to quantitatively understand the color-suppressed decays $\bar{B}^0 \to D^{(*)0}\pi^0$ in perturbative QCD(pQCD) [9] and soft-collinear effective theory (SCET) [10], the model-independent discussions [11–13] can help to get interesting suggestions about the magnitudes and relative phase of the parameters $a_{1,2}$. A couple of important findings can be summarized as follows.

(1) A sizable relative strong interaction phase is expected between class-1 and class-2 $B \to D^*\pi$ decay amplitudes [11].
(2) The parameter $|a_2|$ is extracted to be $|a_2(D\pi)| \sim 0.35$–0.60 and $|a_2(D^*\pi)| \sim 0.25$–0.50 from the data [13].
(3) Several types of possible power corrections to the a_1 parameter have been estimated and found to be small; a

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near-universal value $|a_1| \approx 1.1$ observed experimentally is now put on a firm footing [11].

Now we are not able to give a reliable theoretical interpretation for the first two observations, because of the unknown leading non-perturbative effects involved in the parameter $a_2(D\pi)$. However, the $1/m_b$ power-suppressed non-factorizable contributions to $a_2(D\pi)$, which come from soft exchange between the emitted heavy–light quark pair and the $B\pi$ system, would be expected to be much more important than in the case of $B \to \pi\pi, \pi K$ and thus a reliable estimate of such an effect is crucial.

Earlier discussion on the power-suppressed contribution to $\bar{B}^0 \to D^0\pi^0$ in the QCD light-cone sum rule (LCSR) approach [14] can be found in [15]. Also, in the framework of a generalized QCD LCSR [16] similar effects have been estimated for some of the other important B decays [16–18]. In this paper, we intend to apply the generalized QCD LCSR approach to estimate the soft effect in the color-suppressed $\bar{B}^0 \to D^0\pi^0$ decay and then compare the result yielded with the naive factorization contribution.

This paper is organized as follows: the following section contains a detailed derivation of LCSR's for the power-suppressed soft contribution to the $\bar{B}^0 \to D^0\pi^0$ decay amplitude and the numerical results. The last section is devoted to a discussion and conclusion.

2 LSCRs for soft non-factorizable effect

The relevant effective weak Hamiltonian for the $\bar{B}^0 \to D^0\pi^0$ decay is written as [19]

$$\mathcal{H}_{\rm W} = \frac{G_{\rm F}}{\sqrt{2}} V_{cb}V_{ud}^* \left[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)\right], \quad (1)$$

where $C_{1,2}$ are the Wilson coefficients, V_{ij} the CKM matrix elements and $O_{1,2}$ the four-quark operators given by

$$O_1 = (\bar{c}\Gamma^\mu u)(\bar{d}\Gamma_\mu b), \qquad O_2 = (\bar{d}\Gamma^\mu u)(\bar{c}\Gamma_\mu b), \quad (2)$$

with $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$. Further, by the use of a Fierz transformation (1) can be rewritten as

$$\mathcal{H}_{\rm W} = \frac{G_{\rm F}}{\sqrt{2}} V_{cb}V_{ud}^* \quad (3)$$
$$\times \left[\left(C_1(\mu) + \frac{1}{3}C_2(\mu)\right) O_1(\mu) + 2C_2(\mu)\tilde{O}_1(\mu)\right],$$

where

$$\tilde{O}_1 = \left(\bar{c}\frac{\lambda_a}{2}\Gamma_\mu u\right)\left(\bar{d}\frac{\lambda_a}{2}\Gamma^\mu b\right), \quad (4)$$

with λ_a being the color $SU(3)$ matrices.

Among the non-leading part of the decay amplitude $\langle D^0\pi^0|\mathcal{H}_{\rm W}|\bar{B}^0\rangle$ is the factorizable and power-suppressed soft contribution. We can parameterize it as follows:

$$\mathcal{A}_{\rm NL}(\bar{B}^0 \to D^0\pi^0)$$
$$= \mathcal{A}_F(\bar{B}^0 \to D^0\pi^0) + \mathcal{A}_S(\bar{B}^0 \to D^0\pi^0) \quad (5)$$
$$= -{\rm i}\frac{G_{\rm F}}{2} V_{cb}V_{ud}^* m_B^2 f_D F_0^{B\pi}(m_D^2) a_2^{\rm NL},$$

where $F_0^{B\pi}$ is the $B \to \pi$ form factor, f_D the D meson decay constant, and $\mathcal{A}_F(\bar{B}^0 \to D^0\pi^0)$ and $\mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)$ express the factorizable and power-suppressed soft contributions respectively:

$$\mathcal{A}_F(\bar{B}^0 \to D^0\pi^0)$$
$$= -{\rm i}\frac{G_{\rm F}}{2} V_{cb}V_{ud}^* m_B^2 f_D F_0^{B\pi}(m_D^2)\left(C_1 + \frac{C_2}{3}\right), \quad (6)$$
$$\mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)$$
$$= \frac{G_{\rm F}}{\sqrt{2}} V_{cb}V_{ud}^* (2C_2)\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle_S, \quad (7)$$

and the parameter $a_2^{\rm NL}$ is defined as

$$a_2^{\rm NL} = \left(C_1 + \frac{C_2}{3}\right)\left(1 + \frac{\mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)}{\mathcal{A}_F(\bar{B}^0 \to D^0\pi^0)}\right). \quad (8)$$

For a quantitative estimate of the non-factorizable matrix element $\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle$, we make use of the generalized QCD LCSR method developed in [16]. We start with the correlation function:

$$F_\alpha(p,q,k) = {\rm i}^2 \int {\rm d}^4x {\rm e}^{-{\rm i}(p+q)x} \int {\rm d}^4y {\rm e}^{{\rm i}(p-k)y} \quad (9)$$
$$\times\langle\pi^0(q)|T\{j_\alpha^D(y)\tilde{O}_1(0)j_5^B(x)\}|0\rangle,$$

where $j_\alpha^D = \bar{u}\gamma_\alpha\gamma_5 c$ and $j_5^B = m_b\bar{b}{\rm i}\gamma_5 d$ are currents interpolating the D^0 and $\bar{B}^0$ meson fields respectively. The correlator is a function of the three independent momenta chosen to be q, $p-k$ and k. An important point of this method is to introduce a fictitious unphysical momentum k. Consequently, in the correlator the quark states before and after the b-quark decay will have different four-momentum, and thus one avoids a continuum of light parasitic contributions in the B channel. Of course, the unphysical quantity must disappear from the $\bar{B}^0 \to D^0\pi^0$ ground state contribution in the dispersion integral. This can be guaranteed, as will be seen, by picking out a reasonable kinematical region for which the LCSR calculation is effective.

The kinematical decomposition of the correlation function (9) can proceed in the following form:

$$F_\alpha = (p-k)_\alpha F^{(p-k)} + q_\alpha F^{(q)} + k_\alpha F^{(k)} + \epsilon_{\alpha\beta\lambda\rho}q^\beta p^\lambda k^\rho F^{(\varepsilon)}. \quad (10)$$

Here, the F^i are scalar functions of six independent Lorentz invariants, which are chosen to be $P^2 = (p+q-k)^2$, p^2, q^2, $(p+q)^2$, k^2 and $(p-k)^2$. Using the operator product expansion (OPE) near the light-cone $x^2 \sim y^2 \sim (x-y)^2 \sim 0$, the correlator (9) is calculable. For the calculation to go effectively, the momenta squared P^2, $(p+q)^2$ and $(p-k)^2$ have to be taken spacelike and large in order to stay far away from the hadronic thresholds in both the B and D channels. Furthermore, a simple and possible choice, for the external momentum squared k^2 and kinematical invariant p^2, is to let $k^2 = 0$ and $p^2 = m_D^2$, m_D being the mass of D^0.

The pion is taken on shell and we set $q^2 = 0$. Altogether, the kinematical region used for our LCSR calculation is

$$q^2 = k^2 = 0, \quad p^2 = m_D^2, \quad |(p-k)|^2 \gg \Lambda_{\mathrm{QCD}}, \\ |(p+q)|^2 \gg \Lambda_{\mathrm{QCD}}, \quad |P|^2 \gg \Lambda_{\mathrm{QCD}}. \tag{11}$$

In this region the light-cone OPE is applicable to the correlator (9) and the result can be expressed in the form of hard-scattering amplitudes convoluted with the pion light-cone distribution amplitudes. We note that in (10) the relevant invariant amplitude of our desire is only $F^{(p-k)}$. The QCD result for $F^{(p-k)}$ is, in the general form of a dispersion relation, expressed as

$$F_{\mathrm{QCD}}^{(p-k)}\left((p-k)^2,(p+q)^2,P^2\right) \tag{12}$$
$$= \frac{1}{\pi}\int_{m_c^2}^{\infty} \mathrm{d}s \frac{\mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,(p+q)^2,P^2,p^2)}{s-(p-k)^2}.$$

On the other hand, we can obtain a corresponding dispersion relation on the hadronic level. By inserting in the right hand side of (9) a complete set of hadronic states with D^0 quantum numbers, we get

$$F^{(p-k)}\left((p-k)^2,(p+q)^2,P^2,p^2\right)$$
$$= \frac{\mathrm{i} f_D \Pi\left((p+q)^2,P^2,p^2\right)}{m_D^2-(p-k)^2}$$
$$+ \int_{s_h^D}^{\infty} \mathrm{d}s \, \frac{\rho_h^D(s,(p+q)^2,P^2,p^2)}{s-(p-k)^2}, \tag{13}$$

where $\rho_h^D(s,(p+q)^2,P^2,p^2)$ and s_h^D are respectively the spectral function and the threshold mass squared of the excited and continuum states in the D channel, $\Pi((p+q)^2,P^2,p^2)$ is a 2-point correlation function with the following definition:

$$\Pi\left((p+q)^2,P^2,p^2\right) = \mathrm{i}\int \mathrm{d}^4x \, \mathrm{e}^{-\mathrm{i}(p+q)x} \tag{14}$$
$$\times\langle D^0(p-k)\pi^0(q)|T\{\tilde{O}_1(0)j_5^{(B)}(x)\}|0\rangle.$$

By assuming quark–hadron duality we replace s_h^D with the effective threshold of the perturbative continuum s_0^D, and substitute the hadronic spectral density ρ_h^D in (13) with the corresponding QCD one, i.e.,

$$\rho_h^D(s,(p+q)^2,P^2,p^2)\Theta(s-s_h^D) \tag{15}$$
$$= \frac{1}{\pi}\mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,(p+q)^2,P^2,p^2)\Theta(s-s_0^D).$$

Matching the hadronic relation (13) onto the QCD result (12) yields the expression

$$\frac{\mathrm{i} f_D \Pi\left((p+q)^2,P^2,p^2\right)}{m_D^2-(p-k)^2} \tag{16}$$
$$= \frac{1}{\pi}\int_{m_c^2}^{s_0^D} \mathrm{d}s \frac{\mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,(p+q)^2,P^2,p^2)}{s-(p-k)^2},$$

which then becomes

$$\Pi\left((p+q)^2,P^2,p^2\right) = \frac{-\mathrm{i}}{\pi f_D}\int_{m_c^2}^{s_0^D} \mathrm{d}s \; \mathrm{e}^{(m_D^2-s)/M^2} \tag{17}$$
$$\times \mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,(p+q)^2,P^2,p^2),$$

after a Borel transformation in the variable $(p-k)^2$.

Next, for the above expression which is only valid at large spacelike P^2, we have to perform an analytic continuation to large timelike P^2, keeping the variable $(p+q)^2$ fixed. A natural continuation point is $P^2 = m_B^2$. The analytic continuation of (17) yields the result

$$\Pi\left((p+q)^2,m_B^2,p^2\right) = \mathrm{i}\int \mathrm{d}^4x \; \mathrm{e}^{-\mathrm{i}(p+q)x}$$
$$\times\langle D^0(p-k)\pi^0(q)|T\{\tilde{O}_1(0)j_5^{(B)}(x)\}|0\rangle$$
$$= \frac{-\mathrm{i}}{\pi f_D}\int_{m_c^2}^{s_0^D} \mathrm{d}s \; \mathrm{e}^{(m_D^2-s)/M^2} \tag{18}$$
$$\times \mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,(p+q)^2,m_B^2,p^2).$$

Then we employ the analytical property of the amplitude $\Pi\left((p+q)^2,m_B^2,p^2\right)$ in the spacelike variable $(p+q)^2$. Inserting in the right hand side of (18) a complete set of hadronic states with the $\bar{B}^0$ meson quantum numbers, we have the following dispersion relation:

$$\Pi\left((p+q)^2,m_B^2,p^2\right)$$
$$= \frac{f_B m_B^2 \langle D^0(p)\pi^0(q)|\tilde{O}_1|\bar{B}^0(p+q)\rangle}{m_B^2-(p+q)^2}$$
$$+ \int_{s_h^B}^{\infty} \mathrm{d}s' \, \frac{\rho_h^{(B)}(s',m_B^2,p^2)}{s'-(p+q)^2}, \tag{19}$$

where the B meson decay constant is defined as

$$\langle \bar{B}^0|\bar{b}\mathrm{i}\gamma_5 d|0\rangle = m_B f_B. \tag{20}$$

At this point, we would like to emphasize that the unphysical momentum k disappears from the ground state contribution due to the simultaneous conditions $P^2 = (p+q-k)^2 = m_B^2$ and $(p+q)^2 = m_B^2$, so that the physical $\bar{B}^0 \to D^0\pi^0$ matrix element of the operator $\tilde{O}_1$ is recovered.

Then (18) can be changed to the form of a double dispersion relation as follows:

$$\Pi\left((p+q)^2,m_B^2,p^2\right)$$
$$= -\frac{\mathrm{i}}{\pi^2 f_D}\int_{m_c^2}^{s_0^D} \mathrm{d}s \, \mathrm{e}^{(m_D^2-s)/M^2} \int_{m_b^2}^{R(s,m_B^2,p^2)} \frac{\mathrm{d}s'}{s'-(p+q)^2}$$
$$\times \mathrm{Im}_{s'}\mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,s',m_B^2,p^2). \tag{21}$$

The upper limit R of the integration in s' rests generally on s, m_B^2 and p^2. At present, we make use of quark–hadron duality once more and approximate the integral in (19) by the $s' \geq s_0^B$ part of the dispersion integral (21), where s_0^B

is the effective threshold in the B channel. After a Borel transformation with respect to the variable $(p+q)^2$ has been made, the LCSR for the $\bar{B}^0 \to D^0\pi^0$ matrix element of the operator $\tilde{O}_1$ is of the following form:

$$\langle D^0(p-k)\pi^0(q)|\tilde{O}_1|\bar{B}^0(p+q)\rangle = \frac{-\mathrm{i}}{\pi^2 f_D f_B m_B^2} \times \int_{m_c^2}^{s_0^D} \mathrm{d}s\ \mathrm{e}^{(m_D^2-s)/M^2} \int_{m_b^2}^{\bar{R}(s,m_B^2,p^2,s_0^B)} \mathrm{d}s'\ \mathrm{e}^{(m_B^2-s')/M'^2} \times \mathrm{Im}_{s'}\,\mathrm{Im}_s F_{\mathrm{QCD}}^{(p-k)}(s,s',m_B^2,p^2), \tag{22}$$

where $\bar{R}$ is the upper limit of the integration in s' after the use of the duality ansatz.

In order to obtain a LCSR estimate of the desired soft non-factorizable contribution $\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle_S$, which is due to soft gluon emission off the emitted heavy–light quark pair and subsequent absorption into the $B\pi$ system, we have to know the explicit expression of $F_{\mathrm{QCD}}^{(p-k)}$. In the derivation of $F_{\mathrm{QCD}}^{(p-k)}$, we employ the light-cone expansion form with higher-twist terms included for a massive quark propagator, which, in the fixed-point gauge and only considering a correction of operators with one gluon field, reads [20]

$$S^{ij}(x_1,x_2|m) \equiv -\mathrm{i}\langle 0|T\{q^i(x_1)\,\bar{q}^j(x_2)\}|0\rangle = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k(x_1-x_2)} \times \left\{ \frac{\not{k}+m}{k^2-m^2}\delta^{ij} - \int_0^1 \mathrm{d}v\, g_{\mathrm{s}}\, G_a^{\mu\nu}(vx_1+(1-v)x_2)\left(\frac{\lambda^a}{2}\right)^{ij} \times \left[\frac{1}{2}\frac{\not{k}+m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x_1-x_2)_\mu\gamma_\nu\right]\right\}, \tag{23}$$

where $G_a^{\mu\nu}$ is the gluon-field strength, and g_{s} the strong coupling constant. This means that only the higher-twist components of light-cone distribution amplitudes for the relevant pion, which are corresponding to quark–antiquark–gluon non-local operators, would be involved in the final LCSR result.

After some lengthy calculations we obtain the twist-3 contributions:

$$F_{\mathrm{tw3}}^{(p-k)} = -\frac{m_b f_{3\pi}}{4\sqrt{2}\pi^2} \times \int_0^1 \mathrm{d}v \int \mathcal{D}\alpha_i \frac{\phi_{3\pi}(\alpha_i,\mu)}{m_b^2-(p+q(1-\alpha_1))^2} \times \int_0^1 \mathrm{d}x \frac{x(1-x)}{(1-x)m_c^2 - Q^2x(1-x)} \times q\cdot(p-k)\,[(2-v)q\cdot k + 2(1-v)q\cdot(p-k)] - \frac{m_b f_{3\pi}}{4\sqrt{2}\pi^2}\int_0^1 \mathrm{d}v \int \mathcal{D}\alpha_i \frac{\phi_{3\pi}(\alpha_i,\mu)}{m_b^2-(p+q(1-\alpha_1))^2} \times \int_0^1 \mathrm{d}x \frac{x(1-x)(2x-1)}{m_c^2 - Q^2x(1-x)}\, q\cdot(p-k) \times [(2-3v)q\cdot k + 2(1-v)q\cdot(p-k)], \tag{24}$$

where $\phi_{3\pi}$ is the twist-3 distribution amplitude of the pion, $Q = p-k+qv\alpha_3$. The definition of the twist-3 distribution amplitude as well as of the twist-4 ones $\phi_\perp$, $\phi_{||}$, $\tilde{\phi}_\perp$ and $\tilde{\phi}_{||}$, which will be encountered in the calculation, is given below through the relevant matrix elements:

$$-\sqrt{2}\langle 0|\bar{d}(0)\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta}(vy)d(x)|\pi^0(q)\rangle = \mathrm{i}f_{3\pi}\,[(q_\alpha q_\mu g_{\beta\nu} - q_\beta q_\mu g_{\alpha\nu}) - (q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu})] \times \int \mathcal{D}\alpha_i \phi_{3\pi}(\alpha_i,\mu)\mathrm{e}^{-\mathrm{i}q(x\alpha_1+yv\alpha_3)}, \tag{25}$$

$$-\sqrt{2}\langle 0|\bar{d}(0)\mathrm{i}\gamma_\mu \tilde{G}_{\alpha\beta}(vy)d(x)|\pi^0(q)\rangle = q_\mu \frac{q_\alpha x_\beta - q_\beta x_\alpha}{qx} f_\pi \int \mathcal{D}\alpha_i \tilde{\phi}_{||}(\alpha_i,\mu)\mathrm{e}^{-\mathrm{i}q(x\alpha_1+yv\alpha_3)} + (g_{\mu\alpha}^\perp q_\beta - g_{\mu\beta}^\perp q_\alpha) f_\pi \times \int \mathcal{D}\alpha_i \tilde{\phi}_\perp(\alpha_i,\mu)\mathrm{e}^{-\mathrm{i}q(x\alpha_1+yv\alpha_3)}, \tag{26}$$

$$-\sqrt{2}\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 G_{\alpha\beta}(vy)d(x)|\pi^0(q)\rangle = q_\mu \frac{q_\alpha x_\beta - q_\beta x_\alpha}{qx} f_\pi \int \mathcal{D}\alpha_i \phi_{||}(\alpha_i,\mu)\mathrm{e}^{-\mathrm{i}q(x\alpha_1+yv\alpha_3)} + (g_{\mu\alpha}^\perp q_\beta - g_{\mu\beta}^\perp q_\alpha) f_\pi \int \mathcal{D}\alpha_i \phi_\perp(\alpha_i,\mu)\mathrm{e}^{-\mathrm{i}q(x\alpha_1+yv\alpha_3)}, \tag{27}$$

where $f_{3\pi}$ is a non-perturbative quantity defined by the matrix element $\langle 0|\bar{u}\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta} d|\pi\rangle$, $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\sigma}G^{\rho\sigma}$, $G^{\rho\sigma} = g_{\mathrm{s}}\,\lambda^a/2\, G_a^{\rho\sigma}$, $\mathcal{D}\alpha_i = \mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$, and $g_{\alpha\beta}^\perp = g_{\alpha\beta} - (x_\alpha q_\beta + x_\beta q_\alpha)/qx$. The asymptotic forms of all these distribution amplitudes are given by [21]

$$\phi_{3\pi}(\alpha_i,\mu) = 360\alpha_1\alpha_2\alpha_3^2, \tag{28}$$

$$\phi_\perp(\alpha_i,\mu) = 10\delta^2(\mu)(\alpha_1-\alpha_2)\alpha_3^2, \tag{29}$$

$$\phi_{||}(\alpha_i,\mu) = 120\delta^2(\mu)\epsilon(\mu)(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3, \tag{30}$$

$$\tilde{\phi}_\perp(\alpha_i,\mu) = 10\delta^2(\mu)\alpha_3^2(1-\alpha_3), \tag{31}$$

$$\tilde{\phi}_{||}(\alpha_i,\mu) = -40\delta^2(\mu)\alpha_1\alpha_2\alpha_3, \tag{32}$$

with $\delta^2(\mu)$ and $\epsilon(\mu)$ being two non-perturbative parameters.

By changing the order of the integral variables, (24) is converted into the following form:

$$F_{\mathrm{tw3}}^{(p-k)} = \frac{m_b f_{3\pi}}{16\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^1 \mathrm{d}y \times \int_{x(s,y,P^2)}^1 \frac{\mathrm{d}u}{m_b^2-(p+qu)^2} \times \int_{x(s,y,P^2)}^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v)$$

$$\times\left[s-\frac{m_c^2}{y}+\left((p+q)^2-p^2\right)(2v-x(s,y,P^2))\right]$$
$$+\frac{m_b f_{3\pi}}{16\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y(2y-1)$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{m_b^2-(p+qu)^2}$$
$$\times\int_{x(s,y,P^2)}^{u}\frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v) \quad (33)$$
$$\times\left[\frac{m_c^2}{y}-s+\left((p+q)^2-p^2\right)(2v-3x(s,y,P^2))\right],$$

where $x=\left(s-\frac{m_c^2}{y}\right)/(s-P^2)$.

The derivation of twist-4 contributions turns out to be even more tedious. The result is given as follows:

$$F_{\mathrm{tw4}}^{(p-k)}$$
$$=\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{m_b^2-(p+qu)^2}$$
$$\times\int_{x(s,y,P^2)}^{u}\frac{\mathrm{d}v}{v}\tilde{\phi}_\perp(1-u,u-v,v)\left[3-\frac{2}{v}x(s,y,P^2)\right]$$
$$+\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y(2y-1)$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{m_b^2-(p+qu)^2}$$
$$\times\int_{x(s,y,P^2)}^{u}\frac{\mathrm{d}v}{v}\phi_\perp(1-u,u-v,v)\left[3-\frac{4}{v}x(s,y,P^2)\right]$$
$$+\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{\left[(m_b^2-(p+qu)^2\right]^2}\int_{x(s,y,P^2)}^{u}\frac{\mathrm{d}v}{v^2}\tilde{\Phi}_1(1-u,v)$$
$$\times\left[s-\frac{m_c^2}{y}+\left((p+q)^2-p^2\right)(-v+x(s,y,P^2))\right]$$
$$-\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y(2y-1)$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{\left[(m_b^2-(p+qu)^2\right]^2}$$
$$\times\int_{x(s,y,P^2)}^{u}\frac{\mathrm{d}v}{v^2}\Phi_1(1-u,v)\left[2\left(s-\frac{m_c^2}{y}\right)+(P^2-s)v\right]$$
$$-\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{\left[m_b^2-(p+qu)^2\right]^2}\frac{\tilde{\Phi}_2(u)}{u^2}$$
$$\times\left[s-\frac{m_c^2}{y}+\left((p+q)^2-p^2\right)(-u+x(s,y,P^2))\right]$$
$$-\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{\left[s-(p-k)^2\right]^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y\frac{P^2}{P^2-\frac{m_c^2}{y}}$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{m_b^2-(p+qu)^2}\frac{\tilde{\Phi}_2(u)}{u^2}$$
$$\times\left[\frac{x(s,y,P^2)}{-P^2}\left(2\,q\cdot(p-k)\right)^2\right.$$
$$\left.\times\left(1-\frac{x(s,y,P^2)}{u}\frac{q\cdot k}{q\cdot p}\right)\right]$$
$$+\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{s-(p-k)^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y(2y-1)$$
$$\times\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{\left[m_b^2-(p+qu)^2\right]^2}$$
$$\times\frac{\Phi_2(u)}{u^2}\left[2\left(s-\frac{m_c^2}{y}\right)+(P^2-s)u\right]$$
$$-\frac{m_b^2 f_\pi}{8\sqrt{2}\pi^2}\int_{m_c^2}^{\infty}\frac{\mathrm{d}s}{\left[s-(p-k)^2\right]^2}\int_{\frac{m_c^2}{s}}^{1}\mathrm{d}y(2y-1)$$
$$\times\frac{P^2}{P^2-\frac{m_c^2}{y}}\int_{x(s,y,P^2)}^{1}\frac{\mathrm{d}u}{m_b^2-(p+qu)^2}$$
$$\times\frac{\Phi_2(u)}{u^2}\left[\frac{x(s,y,P^2)}{-P^2}\left(2\,q\cdot(p-k)\right)^2\right.$$
$$\left.\times\left(1-\frac{2x(s,y,P^2)}{u}\right)\left(1-\frac{q\cdot k}{q\cdot p}\right)\right], \quad (34)$$

with the scalar functions Φ_i and $\tilde{\Phi}_i$ defined by

$$\Phi_1(u,v)$$
$$=\int_0^u \mathrm{d}\omega\left(\phi_\perp(\omega,1-\omega-v,v)\right.$$
$$\left.+\phi_{||}(\omega,1-\omega-v,v)\right),$$
$$\Phi_2(u)$$
$$=\int_0^u \mathrm{d}\omega'\int_0^{1-\omega'}\mathrm{d}\omega''\left(\phi_\perp(\omega'',1-\omega''-\omega',\omega')\right.$$
$$\left.+\phi_{||}(\omega'',1-\omega''-\omega',\omega')\right), \quad (35)$$
$$\tilde{\Phi}_1(u,v)$$
$$=\int_0^u \mathrm{d}\omega\left(\tilde{\phi}_\perp(\omega,1-\omega-v,v)\right.$$
$$\left.+\tilde{\phi}_{||}(\omega,1-\omega-v,v)\right),$$
$$\tilde{\Phi}_2(u)$$

$$= \int_0^u \mathrm{d}\omega' \int_0^{1-\omega'} \mathrm{d}\omega'' \Big(\tilde{\phi}_\perp(\omega'', 1-\omega''-\omega', \omega') + \tilde{\phi}_{||}(\omega'', 1-\omega''-\omega', \omega')\Big) . \tag{36}$$

The resulting expression (34) is much more complicated than those in the case of $B \to \pi\pi, \pi K$ [17] and $B \to J/\psi K$ [18], because of the mass asymmetry of the two quarks in the D meson. In contrast to the latter case in which there is no contribution of ϕ_i's due to cancellation in the corresponding twist-4 parts, the twist-4 piece receives the contributions from not only $\tilde{\phi}_i$'s but also ϕ_i's in the present case.

To proceed, we should change the above expressions (33) and (34) into the desired form of a double dispersion relation. For the twist-3 contribution $F^{(p-k)}_{\mathrm{tw3}}$, we have a dispersion expression in $(p-k)^2$:

$$F^{(p-k)}_{\mathrm{tw3}} = \frac{1}{\pi}\int_{m_c^2}^{\infty} \frac{\mathrm{d}s}{s-(p-k)^2} \mathrm{Im_s}\, F^{(p-k)}_{\mathrm{tw3}}(s,(p+q)^2,P^2,p^2) , \tag{37}$$

where

$$\begin{aligned} &\mathrm{Im_s}\, F^{(p-k)}_{\mathrm{tw3}}(s,(p+q)^2,P^2,p^2) \\ &= \frac{m_b f_{3\pi}}{16\sqrt{2}\pi}\int_{\frac{m_c^2}{s}}^1 \mathrm{d}y \int_{x(s,y,P^2)}^1 \frac{\mathrm{d}u}{m_b^2-(p+qu)^2} \\ &\quad\times \int_{x(s,y,P^2)}^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v) \\ &\quad\times \left[s-\frac{m_c^2}{y}+\left((p+q)^2-p^2\right)\left(2v-x(s,y,P^2)\right)\right] \\ &\quad+\frac{m_b f_{3\pi}}{16\sqrt{2}\pi}\int_{\frac{m_c^2}{s}}^1 \mathrm{d}y(2y-1)\int_{x(s,y,P^2)}^1 \frac{\mathrm{d}u}{m_b^2-(p+qu)^2} \\ &\quad\times \int_{x(s,y,P^2)}^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v) \\ &\quad\times \left[-s+\frac{m_c^2}{y}+\left((p+q)^2-p^2\right)\left(2v-3x(s,y,P^2)\right)\right] . \end{aligned} \tag{38}$$

Then for $\mathrm{Im_s}\, F^{(p-k)}_{\mathrm{tw3}}$ we make Taylor expansion in the variable $x(s,y,P^2)$. Up to order $\mathcal{O}(x^3)$ the result is

$$\begin{aligned} &\mathrm{Im_s} F^{(p-k)}_{\mathrm{tw3}}(s,(p+q)^2,P^2) \\ &= \frac{m_b f_{3\pi}}{16\sqrt{2}\pi^2}\int_0^1 \frac{\mathrm{d}u}{m_b^2-(p+qu)^2} \\ &\quad\times \int_{\frac{m_c^2}{s}}^1 \mathrm{d}y \left\{ \int_0^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v) \right. \\ &\quad\times \left[s-\frac{m_c^2}{y}+2v\left((p+q)^2-p^2\right)\right] \\ &\quad-\left[\int_0^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v)\left((p+q)^2-p^2\right) \right. \\ &\quad+\left.\left(s-\frac{m_c^2}{y}\right)\left(\frac{1}{v^2}\phi_{3\pi}(1-u,u-v,v)\right)_{v=0}\right] x(s,y,P^2) \\ &\quad-\left(s-\frac{m_c^2}{y}\right)\left[\frac{\partial}{\partial v}\left(\frac{1}{v^2}\phi_{3\pi}(1-u,u-v,v)\right)\right]_{v=0} \\ &\quad\left.\times\frac{x^2(s,y,P^2)}{2}\right\} \\ &\quad+\frac{m_b f_{3\pi}}{16\sqrt{2}\pi^2}\int_0^1 \frac{\mathrm{d}u}{m_b^2-(p+qu)^2} \\ &\quad\times \int_{\frac{m_c^2}{s}}^1 \mathrm{d}y(2y-1)\left\{\int_0^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v)\right. \\ &\quad\times\left[-s+\frac{m_c^2}{y}+2v\left((p+q)^2-p^2\right)\right] \\ &\quad-\left[3\int_0^u \frac{\mathrm{d}v}{v^2}\phi_{3\pi}(1-u,u-v,v)\left((p+q)^2-p^2\right)\right. \\ &\quad+\left.\left(-s+\frac{m_c^2}{y}\right)\left(\frac{1}{v^2}\phi_{3\pi}(1-u,u-v,v)\right)_{v=0}\right] \\ &\quad\times x(s,y,P^2) \\ &\quad+\left[\left(\frac{4}{v^2}\phi_{3\pi}(1-u,u-v,v)\left((p+q)^2-p^2\right)\right)\right. \\ &\quad-\left.\left(-s+\frac{m_c^2}{y}\right)\frac{\partial}{\partial v}\left(\frac{1}{v^2}\phi_{3\pi}(1-u,u-v,v)\right)\right]_{v=0} \\ &\quad\left.\times\frac{x^2(s,y,P^2)}{2}\right\}+\mathcal{O}(x^3). \end{aligned} \tag{39}$$

With the substitution $u=(m_b^2-p^2)/(s'-p^2)$, the integral in u in the above equation can get back to its dispersion form

$$\int_0^1 \mathrm{d}u \frac{F(u)}{m_b^2-(p+qu)^2} = \int_{m_b^2}^{\infty} \frac{\mathrm{d}s'}{s'-(p+q)^2}\frac{F(u(s'))}{s'-p^2} . \tag{40}$$

At last the desired double dispersion form is achieved.

The twist-4 contribution $F^{(p-k)}_{\mathrm{tw4}}$ in (34) can be treated similarly. The derivation is omitted to save some space. We note that compared with the resulting twist-3 contribution, the twist-4 part has some additional terms containing denominators of the form

$$\frac{1}{[s-(p-k)^2]^2} \quad \text{or} \quad \frac{1}{[m_b^2-(p+uq)^2]^2} . \tag{41}$$

As argued in [18], however, those terms containing higher powers of such denominators are numerically suppressed and can be neglected safely. Therefore in the ensuing discussion we will not take them into account.

Putting everything together, we obtain the final LCSR result for the soft contribution to the matrix element $\langle D^0(p)\pi^0(q)|\tilde{O}_1(0)|\bar{B}^0(p+q)\rangle$:

$$\langle D^0(p)\pi^0(q)|\tilde{O}_1(0)|\bar{B}^0(p+q)\rangle_S = \frac{-\mathrm{i}m_b}{8\sqrt{2}\pi^2 f_D f_B m_B^2}$$

$$
\times \int_{m_c^2}^{s_0^D} \mathrm{d}s \mathrm{e}^{(m_D^2-s)/M^2} \int_{u_0^B}^{1} \frac{\mathrm{d}u}{u} \mathrm{e}^{(m_B^2-(m_b^2-m_D^2(1-u))/u)/M'^2}
\times \int_{\frac{m_c^2}{s}}^{1} \mathrm{d}y \left\{ \frac{f_{3\pi}}{2} \left[\int_0^u \frac{\mathrm{d}v}{v^2} \phi_{3\pi}(1-u,u-v,v) \right.\right.
\times \left(\frac{m_b^2-m_D^2}{u}(2v - x(s,y,m_B^2)) + s - \frac{m_c^2}{y} \right)
- \left(s - \frac{m_c^2}{y} \right) \left(\frac{1}{v^2} \phi_{3\pi}(1-u,u-v,v) \right)_{v=0} x(s,y,m_B^2)
- \left(s - \frac{m_c^2}{y} \right) \left[\frac{\partial}{\partial v} \left(\frac{1}{v^2} \phi_{3\pi}(1-u,u-v,v) \right) \right]_{v=0}
\left. \times \frac{x^2(s,y,m_B^2)}{2} \right]
+ m_b f_\pi \left[\int_0^u \frac{\mathrm{d}v}{v^2} \tilde{\phi}_\perp(1-u,u-v,v) \right.
\times \left(3 - \frac{2}{v} x(s,y,m_B^2) \right)
+ \left(\frac{3}{v^2} \tilde{\phi}_\perp(1-u,u-v,v) \right.
\left.\left.\left. - \left(\frac{1}{v} \frac{\partial}{\partial v} \tilde{\phi}_\perp(1-u,u-v,v) \right) \right)_{v=0} \frac{x^2(s,y,m_B^2)}{2} \right] \right\}
+ \frac{-\mathrm{i} m_b}{8\sqrt{2}\pi^2 f_D f_B m_B^2} \int_{m_c^2}^{s_0^D} \mathrm{d}s \mathrm{e}^{(m_D^2-s)/M^2}
\times \int_{u_0^B}^{1} \frac{\mathrm{d}u}{u} \mathrm{e}^{(m_B^2-(m_b^2-m_D^2(1-u))/u)/M'^2} \int_{\frac{m_c^2}{s}}^{1} \mathrm{d}y(2y-1)
\times \left\{ \frac{f_{3\pi}}{2} \left[\int_0^u \frac{\mathrm{d}v}{v^2} \phi_{3\pi}(1-u,u-v,v) \right.\right.
\times \left(\frac{m_b^2-m_D^2}{u}(2v - 3x(s,y,m_B^2)) - s + \frac{m_c^2}{y} \right)
- \left(-s + \frac{m_c^2}{y} \right) \left(\frac{1}{v^2} \phi_{3\pi}(1-u,u-v,v) \right)_{v=0} x(s,y,m_B^2)
+ \left[4 \frac{m_b^2-m_D^2}{u} \frac{1}{v^2} \phi_{3\pi}(1-u,u-v,v) \right.
- \left. \left(-s + \frac{m_c^2}{y} \right) \frac{\partial}{\partial v} \left(\frac{1}{v^2} \phi_{3\pi}(1-u,u-v,v) \right) \right]_{v=0}
\left. \times \frac{x^2(s,y,m_B^2)}{2} \right]
+ m_b f_\pi \left[\int_0^u \frac{\mathrm{d}v}{v^2} \tilde{\phi}_\perp(1-u,u-v,v) \right.
\times \left(3 - \frac{4}{v} x(s,y,m_B^2) \right)
+ \left(3 \left(\frac{1}{v^2} \tilde{\phi}_\perp(1-u,u-v,v) \right) \right. \quad (42)
\left.\left.\left. + \left(\frac{1}{v} \frac{\partial}{\partial v} \tilde{\phi}_\perp(1-u,u-v,v) \right) \right)_{v=0} \frac{x^2(s,y,m_B^2)}{2} \right] \right\} \Bigg\},
$$

where $u_0^B = (m_b^2 - m_D^2)/(s_0^B - m_D^2)$.

Let us proceed to the numerical discussion. The D channel parameters are taken as [22] $m_D = 1.87\,\mathrm{GeV}$, $m_c = 1.3 \pm 0.1\,\mathrm{GeV}$, $f_D = 170 \pm 10\,\mathrm{MeV}$, $s_0^D = 6 \pm 1\,\mathrm{GeV}^2$ and $M^2 = 1.5 \pm 0.5\,\mathrm{GeV}^2$. The parameters in the B channel are chosen as [23] $m_B = 5.28\,\mathrm{GeV}$, $m_b = 4.7 \pm 0.1\,\mathrm{GeV}$, $f_B = 180 \pm 30\,\mathrm{GeV}$, $s_0^B = 35 \pm 2\,\mathrm{GeV}^2$ and $M'^2 = 10 \pm 2\,\mathrm{GeV}^2$. For the non-perturbative quantities entering the relevant light-cone distribution amplitudes, we use [22] $f_{3\pi} = 0.0026\,\mathrm{GeV}^2$, $\delta^2(\mu_b) = 0.17\,\mathrm{GeV}^2$ and $\epsilon(\mu_b) = 0.36$, whth $\mu_b = \sqrt{m_B^2 - m_b^2} \sim m_b/2 \sim 2.4\,\mathrm{GeV}$. With these inputs, the contributions of twist-3 and -4 fall into the following ranges respectively:

$$
\mathrm{i}\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle_S^{(\mathrm{tw3})} = (0.024\text{–}0.053)\,\mathrm{GeV}^3, \quad (43)
$$

and

$$
\mathrm{i}\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle_S^{(\mathrm{tw4})} = (0.009\text{–}0.017)\,\mathrm{GeV}^3, \quad (44)
$$

and the total contribution reads

$$
\mathrm{i}\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle_S = (0.033\text{–}0.070)\,\mathrm{GeV}^3. \quad (45)
$$

These sum rule results show a good stability against the variations of both the Borel parameters in the given ranges.

3 Discussion and conclusion

Having at hand the LCSR result (45) for the matrix element $\langle D^0\pi^0|\tilde{O}_1|\bar{B}^0\rangle_S$, we can discuss the numerical influence of the power-suppressed soft effect on $\bar{B}^0 \to D^0\pi^0$.

Taking $C_1(\mu_b) = -0.257$, $C_2(\mu_b) = 1.117$, $|V_{cb}| = 0.043$ and $|V_{ud}| = 0.974$, the magnitude of $\mathcal{A}_S$ given by (7) is estimated to be

$$
|\mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)| = (2.48\text{–}5.27) \times 10^{-8}\,\mathrm{GeV}. \quad (46)
$$

It is in order that we now make a numerical comparison between $\mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)$ and the naive factorization piece of the decay amplitude $\mathcal{A}_F(\bar{B}^0 \to D^0\pi^0)$ given by (6). With the LCSR result $F_0^{B\pi}(m_D^2) = 0.30$ [24], we have

$$
R_1 = \mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)/\mathcal{A}_F(\bar{B}^0 \to D^0\pi^0)
= 0.54\text{–}1.15. \quad (47)
$$

This result shows that the resulting soft effect is comparable numerically with the corresponding factorizable one. A similar ratio was estimated for the $B \to J/\psi K$ decay in [18], with the value 0.30–0.70. It seems that power-suppressed soft effects are even more important in $\bar{B}^0 \to D^0\pi^0$ than in $B \to J/\psi K$, as expected.

It is also interesting to compare numerically $\mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)$ with the factorizable contribution of the O_2 operator,

$$
\mathcal{A}_F^{(O_2)}(\bar{B}^0 \to D^0\pi^0) = -\frac{\mathrm{i}}{6} C_2 G_\mathrm{F} V_{cb} V_{ud}^* m_B^2 f_D F_0^{B\pi}(m_D^2).
$$

To this end, we estimate the ratio

$$R_2 = \mathcal{A}_S(\bar{B}^0 \to D^0\pi^0)/\mathcal{A}_F^{(O_2)}(\bar{B}^0 \to D^0\pi^0).$$

The result is $R_2 = 0.17$–0.35. Explicitly, our LCSR calculations favor $a_2^{\rm NL} > 0$, indicating that the correction to a_2, which is relevant to the factorizable and power-suppressed soft part, is positive. This forms a striking contrast to the case of [15] where R_2 is found to be -0.7, a large negative number, so that the sign of $a_2^{\rm NL}$ is predicted to be negative, i.e., a_2 would receive a negative number correction from such non-leading effects.

Naive factorization does not apply for the color-suppressed $\bar{B}^0 \to D^0\pi^0$ decay and as a consequence, the power-suppressed soft exchange correction is expected to be important and is worth discussing carefully, in spite of its non-leading character. We discuss such an effect in the generalized QCD LCSR. The numerical result is in agreement with one's expectations. The size of the resulting contribution to the decay amplitude is found to be comparable with the corresponding factorizable one, about (50–110)% of the latter, and the parameter a_2 would receive a positive number correction, analogously to the case of $B \to J/\psi K$. These observations would be of important phenomenological interests. Of course, at this stage we are not able to go a step further to give a complete estimate of the $\bar{B}^0 \to D^0\pi^0$ decay amplitude, due to the unknown leading non-factorizable soft contributions. More theoretical or phenomenological efforts in this direction are necessary to better understand the color-suppressed charmed decays of B mesons.

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